# Teleparallel Newton–Cartan gravity

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- Geometric formulation: GR $\xrightarrow{c\to\infty}$ Newton–Cartan gravity
  - Galilei-relativistic spacetime geometry
  - Newtonian gravity described by curved connection
- Geometric description of the $c \to \infty$ limit of teleparallel gravity?
  - Here: consider just TEGR
  - Read, Teh (2018): special case of our formalism; null-reduction instead of $c \to \infty$

J Read, NJ Teh: *The teleparallel equivalent of Newton–Cartan gravity*, arXiv:1807.11779, Class. Quantum Gravity **35**, 18LT01 (2018)

# Outline

1. Recap of usual Newton–Cartan gravity
   - Galilei manifolds
   - Newton–Cartan gravity
2. Teleparallel Galilei connections
   - Bargmann structures
   - Teleparallel Newton–Cartan gravity
3. Relation to other theories
   - Teleparallel NC from TEGR
   - Recovering Newtonian gravity

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# Recap of usual Newton–Cartan gravity



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- This talk: mostly assume $\mathrm{d}\tau = 0$ (absolute time)

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- $\rightsquigarrow$ Galilei frame bundle $G(M) \xrightarrow{\pi} M$, principal Gal-bundle

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$$\Gamma^{\rho}_{\mu\nu} = \overset{v}{\Gamma}{}^{\rho}_{\mu\nu} + \frac{1}{2}{T^{\rho}}_{\mu\nu} - {T_{(\mu\nu)}}^{\rho} + \tau_{(\mu}{\Omega_{\nu)}}^{\rho} \tag{2}$$

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  (arbitrary 2-form)
- *Newtonian connection*: $T = 0$ and $\mathrm{d}\Omega = 0$

# Newton–Cartan gravity

## Axioms for Newton–Cartan gravity

1. Spacetime is a Galilei manifold $(M, \tau, h)$ with absolute time, endowed with a Newtonian connection $\widetilde{\omega}$,
2. ideal clocks measure time as defined by $\tau$,
3. free test particles move on timelike geodesics of $\widetilde{\omega}$, i.e. timelike curves $\gamma$ solving

$$\widetilde{\nabla}_{\dot{\gamma}}\dot{\gamma} = 0, \tag{4}$$

4. the field equation

$$\widetilde{R}_{\mu\nu} = 4\pi G\, \rho\, \tau_\mu \tau_\nu \tag{5}$$

holds, where $\rho$ is the mass density.

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- Arises as formal $c \to \infty$ limit of GR
- Coordinate formulation of Newtonian gravity can be recovered (up to possibility of non-absolute rotation)

# Teleparallel Galilei connections



PKS: *Teleparallel Newton–Cartan gravity*, in preparation

# ‘Teleparallelisation’ of NC gravity?

- ‘Teleparallelisation’ GR $\to$ TEGR:
  - Lorentzian manifold $(M, g)$: unique torsion-free metric connection $\overset{\mathrm{L}}{\tilde{\omega}}$
  - $\implies$ General metric connection $\overset{\mathrm{L}}{\omega}$ determined by its torsion $\overset{\mathrm{L}}{T}$
  - $\rightsquigarrow$ Riemannian curvature $\overset{\mathrm{L}}{\tilde{R}}$ expressible purely in terms of $\overset{\mathrm{L}}{\omega}, \overset{\mathrm{L}}{T}, \overset{\mathrm{L}}{R} = 0$
  - $\rightsquigarrow$ Reformulation of GR in terms of $\overset{\mathrm{L}}{\omega}, \overset{\mathrm{L}}{T}$

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  - $\rightsquigarrow$ Reformulation of GR in terms of $\overset{\text{L}}{\omega}, \overset{\text{L}}{T}$
- Analogue for NC gravity?
  - Problem: Galilei connection $\omega$ not uniquely determined by $T$!

# Extending Galilei to Bargmann structures

- Bargmann group $\mathrm{Barg} = \mathrm{Gal} \ltimes (\mathbb{R}^4 \times \mathrm{U}(1))$
- Extend Galilei frame bundle to Barg-bundle $B(M) = G(M) \times_{\mathrm{Gal}} \mathrm{Barg}$

‘Globalised’ construction from:
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- Pulled back to $M$: *extended coframe* $(\mathrm{e}^t = \tau, \mathrm{e}^a, \mathrm{i}a)$
- Transformation under local boost of frame $v \to v - k^a \mathrm{e}_a$:

$$a \to a + k_a \mathrm{e}^a + \frac{1}{2}|k|^2 \tau \qquad (6)$$

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# Extended torsion

- Exterior covariant derivative of $(\tau, \mathrm{e}^a, \mathrm{i}a)$: *extended torsion*

$$\mathrm{d}^\omega(\tau, \mathrm{e}^a, \mathrm{i}a) = (T^t, T^a, \mathrm{i}f) \tag{7}$$

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- For general $\omega$: *Newton–Cartan contortion*

$$\Gamma^{\rho}_{\mu\nu} - \tilde{\Gamma}^{\rho}_{\mu\nu} = \tfrac{1}{2} T^{\rho}{}_{\mu\nu} - T_{(\mu\nu)}{}^{\rho} + \tau_{(\mu} f_{\nu)}{}^{\rho} =: K^{\rho}{}_{\mu\nu} \tag{8}$$

# Teleparallel Newton–Cartan gravity

## Axioms for teleparallel Newton–Cartan gravity

1. Spacetime is a Galilei manifold $(M, \tau, h)$ with absolute time, endowed with a Bargmann structure and a flat Galilei connection $\omega$,
2. ideal clocks measure time as defined by $\tau$,
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$$(\nabla_{\dot\gamma}\dot\gamma)^\rho = K^\rho{}_{\mu\nu}\dot\gamma^\mu\dot\gamma^\nu, \tag{9}$$

4. the field equation

$$-D_\sigma K^\sigma{}_{AB} + D_A K^\mu{}_{\mu B} - K^\mu{}_{\sigma B}T^\sigma{}_{\mu A} + K^\mu{}_{\mu\sigma}K^\sigma{}_{AB} - K^\mu{}_{A\sigma}K^\sigma{}_{\mu B} = 4\pi G\,\rho\,\tau_A\,\tau_B \tag{10}$$

holds, where $\rho$ is the mass density.

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- LHS of (10) is $\widetilde{R}_{AB} \rightsquigarrow$ equivalent to usual NC gravity

# Relation to other theories

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- Field equation of TEGR $\rightsquigarrow$ trace-reverse $\xrightarrow{c\to\infty}$ teleparallel NC field eq.!

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- For $[\mathrm{e}_a, v] = 0$: equation of motion becomes

$$\ddot{\gamma}^a + \omega_c{}^a{}_b \, \dot{\gamma}^c \, \dot{\gamma}^b = -\partial^a \phi \tag{14}$$

# Conclusion

## Summary

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- Teleparallel formulation of NC gravity
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- TNC generalisation ($\mathrm{d}\tau \neq 0$)? $\rightsquigarrow$ probably needs different symmetry algebra ('TNC type II')
- Action formulation?
- Modified teleparallel gravity?
- Theories with more general geometry?
- 'Covariant' post-Newtonian limit of (modified) teleparallel gravity?

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**Many thanks for your attention!**

# Appendix: details

# Gauge fixing the purely spatial torsion

- Purely spatial part of field equation: $\widetilde{R}_{ab} = 0 \overset{\text{3D}}{\Longrightarrow}$ spatial metric flat
- $\implies$ We may assume

$$T^a{}_{bc} = 0 \tag{15}$$

consistently with flatness

Back

# Trautman's 'absolute rotation' condition

- Usual NC: To recover Newtonian gravity, assume $\widetilde{R}^{ab}{}_{\mu\nu} = 0$
- $\iff \exists$ rigid, non-rotating frames
- Such frames in teleparallel NC:

$$\omega_{(ab)} = T_{(ab)t} \quad (16a)$$

$$\omega_{[ab]} = \frac{1}{2} f_{ab} \quad (16b)$$

Back