Local Galilei transformations

[E15] *Local Galilei transformations of dual frame and local connection forms*
Let $(\mathrm{e}_A) = (v, \mathrm{e}_a)$ be a local Galilei frame on a Galilei manifold (M, τ, h). A local Galilei transformation by a (locally defined) Gal-valued function (R, k) is the change of local Galilei frame according to

$$(v, \mathrm{e}_a) \to (\tilde{v}, \tilde{\mathrm{e}}_a) = (v, \mathrm{e}_a) \cdot (R, k)^{-1} = \left(v - \mathrm{e}_b (R^{-1})^b{}_a k^a, \mathrm{e}_b (R^{-1})^b{}_a\right). \tag{1}$$

(a) Determine how the dual frame $(\mathrm{e}^A) = (\tau, \mathrm{e}^a)$ and the local connection form $(\omega^a{}_b, \varpi^a)$ of a Galilei connection change under the above change of frame for a *purely rotational* local Galilei transformation, i.e. with $k = 0$.

Hint: Write the frame change in matrix form. Then use that for a basis change $\tilde{\mathrm{e}}_A = A^B{}_A \mathrm{e}_B$*, the dual basis changes as* $\tilde{\mathrm{e}}^A = (A^{-1})^A{}_B \mathrm{e}^B$*; and that for a connection on a principal bundle whose structure group is a matrix Lie group, local connection forms in general transform under change of local section according to*

$$(\sigma \cdot g^{-1})^* \boldsymbol{\omega} = g \cdot (\sigma^* \boldsymbol{\omega}) \cdot g^{-1} + g \cdot \mathrm{d}(g^{-1}). \tag{2}$$

You should obtain the transformation formulae from proposition 3.16 (i).

(b) Determine how $(\mathrm{e}^A) = (\tau, \mathrm{e}^a)$ and $(\omega^a{}_b, \varpi^a)$ transform under a Milne boost, i.e. a local Galilei boost, i.e. a local Galilei transformation with $R = \mathbb{1}$.

Hint: Proceed as for the rotations. You should obtain the formulae from proposition 3.16 (ii).

[E16] *Deriving Milne boosts of the covariant space metric and the Newton–Coriolis form*
Using the transformation behaviour of (e^A) and ϖ^a, compute how the covariant space metric $\underset{v}{h} = \delta_{ab} \mathrm{e}^a \otimes \mathrm{e}^b$ and the Newton–Coriolis form $\Omega = \delta_{ab} \varpi^a \wedge \mathrm{e}^b$ with respect to v transform under the Milne boost $v \to \tilde{v} = v - k^a \mathrm{e}_a$ parametrised by (k^a). Show that you obtain the behaviour as stated in proposition 1.19.

Hint: To compare to proposition 1.19, note that for the vector field $k = k^a \mathrm{e}_a$*, the corresponding one-form is* $k^\flat = \underset{v}{h}(k, \cdot) = \delta_{ab} k^a \mathrm{e}^b$*. Compute* $\mathrm{d}k^\flat$ *from this expression, and re-express* $\mathrm{d}\mathrm{e}^a$ *in terms of the torsion via proposition 3.14 (ii).*